

Main Steps

There are five main steps for an induction proof.

Step 1: State your $P(n)$. State your $P(n)$, which should be a property as a function of n . Also state for which n you will prove your $P(n)$ to be true.

Step 2: State your base case. State for which n your base case is true, and prove it.

Step 3: State your induction hypothesis. State your induction hypothesis. State your induction hypothesis! Without it, the whole proof falls apart. Usually it is just restating your $P(n)$ (with no restriction on n).

Step 4: Inductive Step. Now consider $P(n + 1)$. This is where you try to prove a larger case of the problem than you assumed in your induction hypothesis. What are you trying to prove? Keep this in mind when you do this step. Remember, use your induction hypothesis somewhere, and clearly state where. If you haven't used your induction hypothesis in this step, then you are not doing a proof by induction. So you'd better need to use it.

Step 5: Conclusion. This is optional. You can re-state the problem.

Comments

- If your $P(n)$ doesn't mention n in it anywhere, there's trouble.
- $P(n)$ is a *property*, not a number, so you *cannot* manipulate it mathematically, like $P(n) = 5$, or $P(n + 1) < P(n)$.
- Be careful with the base case...sometimes you will need more than one, as with some recurrence relations.
- Recall the difference between strong and weak induction. They're equivalent, but sometimes using one is easier than using the other.
- When trying to prove some equation holds, that is, if you are trying to prove that $x = y$ for some x and y , please do not start with assuming they are equal and then modifying both sides of the equations until you get an equation that is actually true. For example:

$$\begin{aligned}x &= y \\0 \times x &= 0 \times y \\0 &= 0.\end{aligned}$$

Obviously I can 'prove' that $x = y$ using this method for any x and y , regardless of whether or not x and y are actually equal. What is better is to start with x , make modifications to x through a string of equalities that somehow ends with y . That is, $x = \dots = y$.

Example

For $n \geq 0$, let $P(n)$ be the statement that $\sum_{i=0}^n 2^i = 2^{n+1} - 1$. We want to show that $P(n)$ is true for all $n \geq 0$.

Base Case: We want to show that $P(n)$ is true for $n = 0$; that is, that $\sum_{i=0}^0 2^i = 2^1 - 1$. Fortunately,

$$\begin{aligned} LHS &= \sum_{i=0}^0 2^i \\ &= 2^0 \\ &= 1 \\ &= 2 - 1 \\ &= 2^1 - 1 = RHS . \end{aligned}$$

Induction Hypothesis: Assume $P(n)$ is true for some $n \geq 0$. That is, assume that

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1 .$$

Inductive Step: With the assumption that $P(n)$ is true for some n , we want to prove that $P(n+1)$ is true, that is, we want to prove that *if* $P(n)$ is true *then* $P(n+1)$ is also true. Specifically, to show $P(n+1)$ is true, we need to prove $\sum_{i=0}^{n+1} 2^i = 2^{n+2} - 1$.

$$\begin{aligned} LHS &= \sum_{i=0}^{n+1} 2^i \\ &= 2^{n+1} + \sum_{i=0}^n 2^i \\ &\stackrel{IH}{=} 2^{n+1} + (2^{n+1} - 1) \\ &= 2 \cdot 2^{n+1} - 1 \\ &= 2^{n+2} - 1 = RHS . \end{aligned}$$

Conclusion: By mathematical induction, we have shown that for all $n \geq 0$, $\sum_{i=0}^n 2^i = 2^{n+1} - 1$.
