

Main Steps

The 3 main steps of a proof by construction are as follows:

Step 1: State your claim. State the specific type of object you claim exists.

Step 2: Construct the object. Given any set of valid parameters, show how to construct an object of the desired type.

Step 3: Conclusion. This is optional. You can re-state the goal.

Comments

- This proof technique is a little less structured, as it is more dependent on the nature of the statement you are trying to prove.

Example

Suppose we want to show that for even $n > 2$, there is an n -node 3-regular graph.

We prove this by constructing such a graph given any valid parameter n . For any even $n > 2$, we construct a 3-regular graph $G = (V, E)$ as follows. We set

$$V = \{v_0, v_1, \dots, v_{n-1}\},$$

and

$$E = \{\{v_i, v_{i+1}\} \mid i \leq n - 2\} \cup \{v_0, v_{n-1}\} \cup \{\{v_i, v_{i+\frac{n}{2}}\} \mid i \leq \frac{n}{2} - 1\}.$$

You can see this visually by drawing the nodes equally spaced around a circle (which achieves 2-regularity), then drawing chords between vertices opposite each other on the circle. The graph constructed is well-defined and 3-regular.

Thus for any even $n > 2$, there is a 3-regular graph on n nodes.
