Main Steps

The 4 main steps for proving a language A is *not* regular is as follows:

Step 1: Demon Picks $k \geq 0$. You are given some pumping length $k \geq 1$.

Step 2: You pick xyz. Select x, y, z such that $xyz \in A$ and $|y| \geq k$.

Step 3: Demon Picks Decomposition u, v, w. The demon picks u, v, w such that $y = u, v, w$ and $v \neq \epsilon$.

Step 4: You pick $i \geq 0$. Construct a string xuv^iwz that is not in A, for some $i \geq 0$. Remember that you may want to set i to 0 in order to accomplish this.

Comments

- The pumping lemma states a property of regular languages. You cannot use it to prove a language is regular, but you can use its contrapositive to prove a language is *not* regular. As a reminder, here is the pumping lemma in its positive form:

  > If a language A is regular, then there exists a $k \geq 1$ such that for all strings x, y, z with $xyz \in A$ and $y \geq k$, there exist strings u, v, w with $y = uvw$ and $v \neq \epsilon$, and for all $i \geq 0$, $xuv^iwz \in A$.

- Be sure your string xyz is in A.
- Be sure to handle all possible decompositions of the string y as uvw. The demon is picking this decomposition, and you cannot pick which decomposition he chooses.
- Don't choose an i that is fractional or negative! This is not allowed by the statement of the pumping lemma; i must be an integer ≥ 0.
- Your string xyz should somehow depend on the pumping length k. If it doesn't depend on k, then you cannot guarantee that it will be long enough for all possible values the demon provides.

Example

Consider the language $A = \{a^n b^n \mid n \geq 0\}$. We will show that A is not regular using the pumping lemma.

Proof. By way of contradiction, suppose that A is regular. Then there is some pumping length $k \geq 0$ chosen by the demon. Now we take $x = \epsilon$, $y = a^k$, and $z = b^k$. Then $xyz = a^k b^k \in A$, and $|y| = k$. The demon must now pick u, v, w such that $y = uvw$ and $v \neq \epsilon$. Say the demon picks u, v, w of lengths j, m, n, respectively. Then $k = j + m + n$ and $m > 0$. But whatever the demon picks, we can win by taking $i = 2$:

$$xuv^2wz = a^j a^{2m} a^n b^k = a^k a^m k a^k,$$

which is not in A because there are different numbers of a's and b's.