Trees

See Chapter 8 of the text
Trees are the second-most important data structure in programming, following only lists.

We will think about trees in two different ways.
Graph definition: A tree is a kind of directed graph, so it has a set of nodes and a set of edges connecting the nodes. There is one special node called the **root**. Each node except for the root in a tree has an incoming edge from one other node. The root has no incoming edges. There is a path of edges from the root to every other node. Nodes that have no outgoing edges are called **leaves**.

![Tree Diagram]

We call the node at the start of an edge a **parent** node. The node at the end of this edge is the parent's **child**. In this terminology parents can have multiple children, but children have exactly one parent.
We measure the **length** of a path in the tree by the number of edges it contains (not the number of nodes). The **height** of a node is the longest path from it to a leaf. The height of the overall tree is the height of its root. The **depth** of a node is the length of the path from it to the root. The depth of the root itself is 0.

<table>
<thead>
<tr>
<th>Node</th>
<th>Height</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>
Here is a recursive definition of a tree:

A tree is either empty or it is a root $r$ and 0 or more non-empty subtrees connected to $r$ by an edge.
A binary tree is one in which each node can have at most two children. The children are often referred to as the *left* and *right* children.
Trees are used in many situations:

- Anything hierarchical, like file systems or administrative structures or Java class structures can be represented by trees.
- Whenever a program is compiled the compiler builds a tree representation of the program, guided by a grammar for the programming language.
- Games are often represented by trees, where the root represents the current state of the game and children represent possible moves.
- Indexes in a database are built on tree structures.
Since arithmetic operators take two arguments, one use of binary trees is in representing expressions. You might represent $3*(4+5)$ as

```
        *
       /  \
      3  +
     /   /
    4   5
```

Such a tree has operators in the interior nodes and numbers in the leaves. There is an easy recursive algorithm to compute its value -- evaluate the left child, evaluate the right child and apply the operator to those values.
So how do we represent trees?

You can use arrays. If there are at most 2 children of each node, you can put the root at index 0, its children at index 1 and 2, the children of the node at index 1 could be at indices 3 and 4, and so forth. The children of the node at index \( n \) are at index \( 2n+1 \) and \( 2n+2 \). The parent of node at index \( k \) is at index \( (k-1)/2 \). So the array [2 9 1 3 5] represents the tree

```
  2
 /|
/ |
9 1
 |
3 5
```

If a node could have 3 children the kids of node \([n]\) would be at \([3n+1]\) \([3n+2]\) and \([3n+3]\)
A more flexible scheme is to use a linked structure. In Lab 5 we will use the following 3 classes:

```java
abstract class BinaryTree<T> {
    // methods we want the tree classes to have
}

class EmptyTree<T> extends BinaryTree<T> {
    // no data and just trivial methods
}

class ConsTree<T> extends BinaryTree<T> {
    T data;
    BinaryTree<T> left;
    BinaryTree<T> right;
    // non-trivial methods
}