Map and Apply
Map and Apply are two very powerful Scheme tools that are frequently misunderstood by students.

Map in general can take a function of n-arguments and n lists, but it is easier to think of it if we have a function of one argument and a single list of values. The result of f

(map f lat)

is a new list, whose first element is (f (car lat)), whose second element is (f (cadr lat)) and so forth. The ith element of the returned list is the result of applying f to the ith element of lat.
For example,

$$(\text{map } (\lambda (x) (+ x 2)) \ '{(1 \ 2 \ 3 \ 4 \ 5)})$$
returns

$$(3 \ 4 \ 5 \ 6 \ 7)$$

The second argument to map does not need to be a flat list; map takes as an argument each element at the top level of the list.

For example,

$$(\text{map } \text{car} \ '(({1 \ 2}) \ {(3 \ (4 \ 5))} \ (6)))$$
returns

$$(1 \ 3 \ 6)$$
Map in general can take a function of n arguments and n argument-lists, all of the same length. The result of

\[(\text{map } f \text{ arg1 arg2 ... argn })\]

is a new list whose ith element is the result of applying f to the ith element of each of the argument lists

For example

\[(\text{map (lambda (x y) (+ x y)) '(1 2 3) '(4 5 6)})\]

returns

\[(5 7 9)\]
Map has all kinds of useful applications. For example, suppose we have a binding list in a let expression:

\[
([x \ 3] \ [y \ 45] \ [z \ 123])
\]

We can get the list of symbols being bound, \( (x \ y \ z) \), from

\[
\text{(map car '([x 3] [y 45] [z 123]))}
\]

and the list of values being bound from

\[
\text{(map cadr '([x 3] [y 45] [z 123]))}
\]
If you write the factorial function

```scheme
(define fact
  (lambda (x)
    (if (= x 1) 1 (* x (fact (- x 1))))))
```

and what to check it out quickly, you can do so with

```scheme
(map fact '(1 2 3 4 5 6 7))
```

and get

```scheme
(1 2 6 24 120 720 5040)
```
Apply has a simpler definition, but I find that students have a harder time thinking about it. If \( f \) is a function of \( n \) arguments and \( L \) is a list of \( n \) elements,

\[
\text{(apply } f \text{ } L) \\
\]

is the result of calling \( f \) with the elements of \( L \) as its arguments.

For example, \((+ \ '(2 3))\) makes no sense but \(\text{(apply } + \ '(2 3))\) does make sense and has the value 5, as you would expect.
We can define a procedure that finds the distance of a 2D point from the origin:

```
(define dist
  (lambda (x y)
    (sqrt (+ (* x x) (* y y))))))
```

(dist 3 4) correctly returns 5.

However, if we have a point p defined as a pair: (x y) we can't use dist to find its distance from the origin because dist wants 2 separate arguments. However we can do this with apply:

```
(apply dist p)
```
Max is a pre-defined Scheme function that takes any number of numerical arguments and returns the largest of its arguments. For example,

```scheme
(max 2 5 6 3 9 5 6)
```
returns 9.

We might want to find the maximum value of a lat; we can get this with

```scheme
(apply max lat)
```
Map and apply are often used together to recurse on a structured list.

For example, here is a function that finds the largest number in a structured list of numbers, such as (2 (4 5 (6)) 3 (4 (5))):

```
(define largest
  (lambda (L)
    (cond
      [(null? L) -1]
      [(number? L) L]
      [else (apply max (map largest L))])))
```
Here is a function that counts the number of atoms in an S-expression. Remember that an S-expression can be null, an atom, or a list:

```scheme
(define count
  (lambda (L)
    (cond
     [(null? L) 0]
     [(not (pair? L)) 1] ; this means L is an atom
     [else (apply + (map count L))])))