CS 357
Transformations

The View Transformation

\[
V = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-a_v & -b_v & -c_v & 1
\end{bmatrix} \times \begin{bmatrix}
\frac{a_x}{r} & -\frac{b_x}{r} & 0 & 0 \\
\frac{b_x}{r} & \frac{a_x}{r} & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \times \begin{bmatrix}
r/R & 0 & -\frac{c_x}{R} & 0 \\
0 & 1 & 0 & 0 \\
\frac{c_x}{R} & 0 & r/R & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{b_z a_x - a_z b_x}{h} & 0 & 0 \\
\frac{a_z b_x - b_z a_x}{h} & \frac{c_z R}{h} & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

where \( r = \sqrt{a_x^2 + b_x^2} \), \( R = \sqrt{a_x^2 + b_x^2 + c_x^2} \), and \( h = r \sqrt{a_z^2 + b_z^2 + c_z^2} \)

The Perspective Transformation

\[
P = \begin{bmatrix}
D & 0 & 0 & 0 \\
0 & D & 0 & 0 \\
0 & 0 & \frac{Y}{Y-H} & 1 \\
0 & 0 & \frac{-HY}{Y-H} & 0
\end{bmatrix}
\]

where \( H, D, \) and \( Y \) are distances to the Hither plane, viewplane and Yon plane.

The Window Transformation

\[
W = \begin{bmatrix}
\frac{w_R - w_L}{v_R - v_L} & 0 & 0 & 0 \\
0 & \frac{w_T - w_B}{v_T - v_B} & 0 & 0 \\
0 & 0 & \frac{w_T v_R - v_L w_R}{v_T - v_B} & 1 \\
\frac{v_R - v_L}{v_T - v_B} & \frac{w_T w_v - v_L w_T}{v_T - v_B} & 0 & 1
\end{bmatrix}
\]

where the \( v \)'s and \( w \)'s are respectively the boundaries of the viewport on the viewplane and the window on the screen.
The full view pipeline, which transforms object coordinates into screen coordinates, is \([h, v, z, l] = [x, y, z, l]VPW <\text{homogenize.} > = [x, y, z, l]VP <\text{homogenize} > W\)

**Example:** Suppose the viewer is at \((10, 10, 10)\) looking at point \((0,0,0)\), with the viewer's up-vector <0,0,1>. The viewplane is \(z=10\) (i.e, \(D=10\)); the hither and yon planes are defined by \(Y=15\) and \(H = 5\). We use a view angle of \(\Theta=40\) degrees, and use window boundaries 0 to 500 in each direction. Where on the screen is the point \((2,5, 1)\) (specified in world coordinates) drawn?

\[
(a_v, b_v, c_v) = (10, 10, 10) \quad <a_z, b_z, c_z> = <-10, -10, -10>
\]
\[
<V> = \begin{bmatrix}
-0.707 & -0.408 & -0.578 & 0 \\
0.707 & -0.408 & -0.578 & 0 \\
0 & 0.818 & -0.578 & 0 \\
0 & 0 & 17.33 & 1
\end{bmatrix}
\]

We are given that \(D = 10\), \(Y = 15\) and \(H = 5\), so \(P = \begin{bmatrix} 10 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 0 & 1.5 & 1 \\
0 & 0 & -7.5 & 0 \end{bmatrix}\)

\(v_L = -D\tan(40) = -8.39\) Similarly, \(v_R = v_T = 8.39\), \(v_B=-8.39\).

The window boundaries are given as \(w_L=0, w_R = 500, w_T = 0, w_B = 500\).

This gives \(W = \begin{bmatrix} 29.79 & 0 & 0 & 0 \\
0 & -29.79 & 0 & 0 \\
0 & 0 & 1 & 0 \\
250 & 250 & 0 & 1 \end{bmatrix}\)

\([2 \quad 5 \quad 1]VPW = [2.12 \quad -2.04 \quad 12.71 \quad 1]PW = [21.2 \quad -20.4 \quad 11.56 \quad 12.71]W\)

This last result homogenizes to \([299.7, 297.8, 0.91, 1]\). Note that the first two coordinates are within our window boundaries and the z-coordinate is between 0 and 1. We would draw the point at pixel \((300, 298)\).