Language Definitions and Notations
Here are a bunch of basic definitions that we will use all semester. There is nothing exciting here but you won't be able to follow much until you get these in your head.

$\Sigma$ is a finite set of symbols called our \textit{alphabet}. This could be the set \{0,1\} of binary digits, or the set of lower-case letters 'a' to 'z'. Don't let the term "alphabet" confuse you. $\Sigma$ could also be the set of valid Java keywords and identifiers up to length 64 (so it is finite). Any finite set of atomic elements will do.

A \textit{string} or \textit{word} over $\Sigma$ is any finite sequence of elements of $\Sigma$.

$\varepsilon$ represents the \textit{empty string}: the string of length 0
$\Sigma^n$ is the set of strings over $\Sigma$ of length $n$ (exactly $n$).

$\Sigma^*$ is the set of all strings over $\Sigma$, including the empty string.

$\Sigma^+$ is the set of all strings with positive length over $\Sigma$.

Obviously, $\Sigma^* = \Sigma^+ \cup \{\varepsilon\}$

A language over $\Sigma$ is any subset of $\Sigma^*$. 
Question 1: How big is $\Sigma^*$?
   Well, if $\Sigma$ is the empty set then $\Sigma^*$ is \{\varepsilon\}. If $\Sigma$ is not empty then $\Sigma^*$ is countable -- it is a countable union of finite sets.

Question 2: How many languages are there over $\Sigma$?
   If $\Sigma$ is empty there are two, both trivial: \{\} and \{\varepsilon\}. If $\Sigma$ is not empty there are uncountably many languages over it (for if you could number the subsets of $\Sigma^*$ you could create a new subset that wasn't any of them: if string $i$ is in subset $i$ then exclude it from your new subset, otherwise include it).