1. Find a DFA that accepts the language of strings of 0s and 1s in which the number of 0s and the number of 1s are either both even or both odd. For example, 00100100 (both are even) and 111000 (both are odd) are both strings in this language while 00111 is not.

In the following DFA “EO” represents the state where we have seen an even number of 0s and an odd number of 1s; the other states are similar.

You can get an even simpler DFA if you notice that this language is just the language of strings with even length.
2.
   a. Convert the following $\varepsilon$-NFA into a DFA.
   b. Describe in English or with a regular expression the strings that are accepted by this automaton. This should be easy after you find the DFA.

\[\text{Diagram of NFA}\]

b) It is pretty easy to see what strings this DFA accepts: $00^*1$, $00^*11$, 1, 11. You can put all of those together as $0^*1+0^*11$ or $00^*1(1+\varepsilon)$.  

\[\text{Diagram of DFA}\]
3. Here is a DFA:

![Diagram of DFA]

I want to convert this DFA into a regular expression using the algorithm we developed in class. Here are the first two columns of a table of the $r_{ij}^k$ expressions.

<table>
<thead>
<tr>
<th></th>
<th>$k=0$</th>
<th>$k=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{11}^k$</td>
<td>$\varepsilon$</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$r_{12}^k$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$r_{13}^k$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$r_{21}^k$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$r_{22}^k$</td>
<td>$\emptyset + \varepsilon$</td>
<td>0 + $\varepsilon$</td>
</tr>
<tr>
<td>$r_{23}^k$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$r_{31}^k$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$r_{32}^k$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$r_{33}^k$</td>
<td>$\varepsilon$</td>
<td>$\varepsilon$</td>
</tr>
</tbody>
</table>

What is the $r_{23}^2$ entry?

Our recursive formula was $r_{ij}^k = r_{ij}^{k-1} + r_{ik}^{k-1} (r_{kk}^{k-1})^* r_{kj}^{k-1}$

Setting $i=2$, $j=3$, $k=2$ gives $r_{23}^2 = r_{23}^1 + r_{22}^1 (r_{22}^1)^* r_{23}^1$

$$= 1 + (0 + \varepsilon)(0 + \varepsilon)^* 1$$

$$= 0^* 1$$
4. Which of the following languages are regular? You don’t need to prove your answer. Write an “R” in the blank next to the description of each language you think is regular. Write “N” for any language you think is not regular. In each case the alphabet is \( \Sigma = \{0,1\} \)

a. _R__Strings where there are not two consecutive 1’s, such as 01001 or 000. It is easy to write a DFA for this.

b. _N____Strings of odd length with 1 in the center, such as 0001000 or 1101100. Consider \( 0^n10^n \). If \( n \) is big enough this can be pumped into something that doesn’t have 1 in the center.

c. _R____Strings of the form \( \alpha \beta \) where \( |\alpha| = |\beta| \) such as 000111. These are just strings of even length.

d. _R__Strings that start and end on the same digit, such as 1010101 and 0001110
\[ 0(0+1)^*0 + 1(0+1)^*1 \]

e. _R__Strings whose digits sum to no more than 10000.
Make a DFA with a 1 transition from Start to Q1, a 1-transition from Q1 to Q2, a 1 transition from Q2 to Q3, etc, all the way up to Q10000. Each state has a 0-transition to itself. Q10000 is final.
5. Consider the language \( \{0^n1^m \mid n > m \geq 0 \} \). Strings in this language have any positive number of 0s followed by fewer 1s. Give a careful pumping lemma argument that this language is not regular.

Suppose this language is regular; let \( N \) be its pumping constant. Consider the string \( w=0^N1^{N-1} \). That string is in our language. Consider any decomposition \( w=xyz \) with \( |xy|<N \) and \( y \) not empty. Then \( y \) consists of a positive number of 0s. So \( xy^0z \) has fewer 0s and just as many 1s as \( w \), so it does not have more 0s than 1s. This means \( xy^0z \) is not in our language. So our string \( w \) is not pumpable, and this violates the Pumping Lemma. Since the Pumping Lemma holds for all regular languages, our language isn’t regular.
6. Suppose language $L$ is regular. Is $L^2 = \{xx \mid x \text{ is in } L\}$ necessarily regular? Either prove that it is or give a language $L$ where $L^2$ isn’t regular. For example, if $L = \{01, 110, 0\}$ then $L^2$ is $\{0101, 110110, 00\}$.

This was the central part of a homework problem. Let $L$ be the language described by $0^*1$. Suppose $L^2$ is regular; let $N$ be its pumping constant. Then $w = 0^N10^N1$ is a string in $L^2$. But this string isn’t pumpable: if $w = xyz$ where $|xy| < N$ and $y$ is not empty, then $y$ has a positive number of 0s, and $xy^2z$ has more 0s in its first half than its second half, so it doesn’t have the form $XX$ where $X$ is in $L$. 
7. Give a context-free grammar for the language \( \{ 0^n1^m0^n1^m \mid n>0, m>0 \} \). Read the superscripts carefully. Strings in this language have the same number of initial 0s and trailing 1s, and in the middle they have the same number of 1s and 0s. For example 0001100111 (also written as \( 0^31^20^21^3 \)) is in this language.

\[
S \rightarrow 0 \, S \, 1 \mid T \\
T \rightarrow 1 \, T \, 0 \mid 10
\]

Actually, that allows \( n \) (the first and last index) to be 0. If that bothers you change the first rule to

\[
S \rightarrow 0 \, S \, 1 \mid 0 \, T \, 1
\]