There are 6 numbered questions. The 6 parts of Question 1 are worth 4 points each. Questions 2 through 6 are worth 15 points each. You get one point for free.

1. Which languages are regular? You don’t need to prove your answers. Write an “R” in the blank next to the description of each language you think is regular. Write “N” for any language you think is not regular. In each case the alphabet is $\Sigma = \{0, 1\}$

   a. _____Strings that end in exactly 10 1’s. So 010111111111 is in this language but 011111111111 is not.

   b. _____Strings with any number of 0’s followed by an even number of 1’s.

   c. _____Strings where the digits sum to a number divisible by 5.

   d. _____Strings where there are at least as many 0’s as 1’s.

   e. _____$0^*\mathcal{L}$ (that is the concatenation of two languages), where $\mathcal{L} = \{0^n \mid n \text{ is prime} \}$

   f. _____Strings of length 1000 that have a prime number of 1’s.
2. Here is an \(\varepsilon\)-NFA, with start state A.
   a) Convert this NFA to a DFA
   b) Describe in English the strings it accepts.
3. Suppose we know that for some language \( \mathcal{L} \) we know that the language
\[
00\mathcal{L} = \{00\alpha \mid \alpha \in \mathcal{L}\}
\]
is regular. Must \( \mathcal{L} \) be regular? Either give an example where \( \mathcal{L} \) is not regular and \( 00\mathcal{L} \) is regular, or else show that \( \mathcal{L} \) must be regular if \( 00\mathcal{L} \) is.
4. Consider the following DFA. We had an algorithm for converting a DFA to a regular expression. This involved making a table of regular expressions \( r_{ij}^k \).

![DFA Diagram]

Here is the first column of a table of the \( r_{ij}^k \) expressions; find the 4 entries of the second column.

<table>
<thead>
<tr>
<th></th>
<th>( k=0 )</th>
<th>( k=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r^k_{11} )</td>
<td>( \epsilon + 1 )</td>
<td></td>
</tr>
<tr>
<td>( r^k_{12} )</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>( r^k_{21} )</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( r^k_{22} )</td>
<td></td>
<td>( \epsilon + 0 )</td>
</tr>
</tbody>
</table>
5. Use the pumping lemma to show carefully that the language \( \{0^m1^n0^n | m \geq 2, n \geq 0\} \) is not regular.
6. Give a grammar for the language \{0^n1^m \mid n > m > 0 \}
This page is extra space. If you want me to grade anything here indicate that clearly.

Please write and sign the Honor Pledge when you have finished the exam.