The exam has 6 questions. #1 is worth 20 points; the other five are worth 16 points each

1. Here is a list of languages. Circle the indices (a, b, etc.) for the ones that are context-free. You don’t need to justify your answers. Remember that regular languages are also context-free.

   a. Numerical strings whose digits sum to an even number, such as 24473 or 112233.

   b. \{0^n1^n0^m1^m \mid n>0 \text{ and } m>0\}

   c. \{0^n1^n \mid n > 0 \text{ and } n \text{ is odd}\}

   d. Strings of 0s and 1s with odd length that have a 1 as the center digit. For example, 00100, 11111, and 01101 are all in this language.

   e. Strings of the form vcw, where v and w are both strings of 0s, 1s, and 2s (and c is just the letter c), such that the digits of v sum to the same value as the digits of w. For example, 012011c221 is such a string because the digits before and after c both sum to 5.
2. Here is a grammar. Draw a parse tree for the derivation of bbaaa (don’t misread that; it is $b^2a^3$)

   A $\Rightarrow$ Aa $|$ B
   B $\Rightarrow$ BB $|$ A $|$ b
3. Construct a PDA for the language \( \{ 1^k2^m3^n \mid k, m, n > 0, m=k+n \} \). For example, \(1^22^73^5\) is in this language.
4. Convert this grammar to Chomsky Normal Form:
   A => AB | AC | 0
   B => 1B | ε
   C => AB | 2 | ε
Consider the following PDA, which accepts by empty stack. We had an algorithm for converting a PDA into a grammar. Give a derivation in this grammar for the string 001011, which is accepted by this PDA.
6. Give a careful proof that \( \{ 1^n 2^n 3^m \mid n>0, m>n \} \) (i.e., same number of 1s and 2s, more 3s) is not context-free.
Please write and sign the Honor Pledge when you have finished the exam.