CS 150: Intro to Computer Science	**Mathematical Self-Assessment**

CS 150 does not have a formal mathematics requirement, and does not require knowledge of calculus. That said, the course does require a degree of comfort working with mathematical notation, logic, and concepts that some students find intimidating. Students who consider themselves math-phobic may wish to consider CS 140, which provides a gentler introduction to programming.

To help gauge your own preparedness, here are a few problems to try puzzling out on your own. These represent the kind of reasoning that'll be required of you in CS 150. It's ok if you struggle with these, but they are the type of problems you'll be expected to work on independently. If this makes you really nervous, you may want to think about taking CS 140 or a math course first.

Exercises

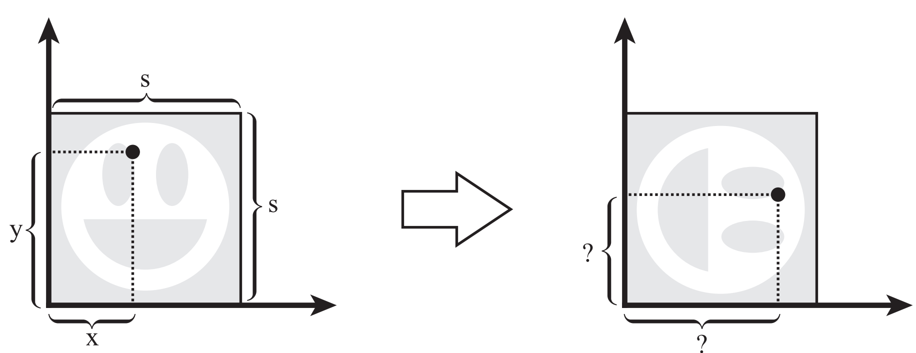
(1) The Fibonacci numbers are a sequence of integers denoted $f_1, f_2, f_3, \ldots$. The first two Fibonacci numbers have a value of 1. That is, $f_1 = 1$ and $f_2 = 1$. Each subsequent Fibonacci number is defined to be the sum of the two that precede it. Symbolically, we'd say that for $i \geq 3$, we define

$$f_i = f_{i-1} + f_{i-2}.$$

What are the next four Fibonacci numbers (f_3, f_4, f_5, and f_6)?

{*Self-check: your answers should sum to 18.*}

(2) Imagine you have a square whose side length is s inches placed in the plane such that its lower-left corner lies at the origin. A black dot is drawn on the square at some position (x, y). Consider rotating the square 90 degrees clockwise in place (rotating around the center of the square, not the origin), as shown below. What are the new coordinates of the dot (in terms of x, y and s)?



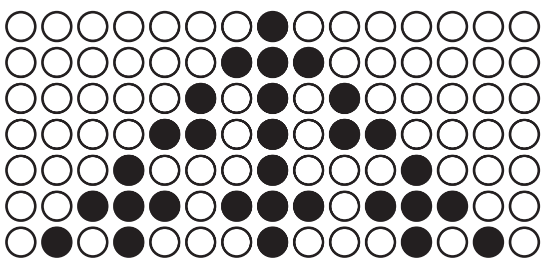
{*Self-check: the answer should include all three variables, each exactly once.*}

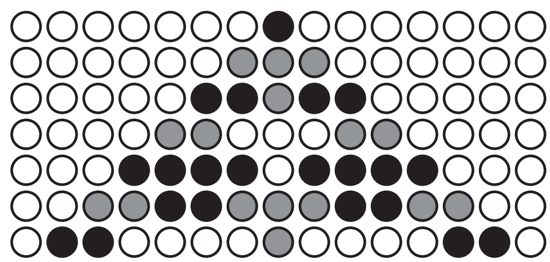
(3) The expression "x factorial", denoted as $x!$, represents the product of all integers from x down to 1. For example, $3! = 3 \cdot 2 \cdot 1 = 6$ and $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 4 \cdot 3! = 24$. Simplify the following expression:

$$\frac{1}{k! \cdot (x-k)!} + \frac{1}{(k-1)! \cdot (x-k+1)!}$$

{*Hint: note that* $k! = k \cdot (k-1)!$ *Use this observation to find a common denominator.*}

(4) Consider the following two patterns:





The pattern on the left is generated as follows: start with a single black dot. On the next row, for each position, compare the above-left dot with the above-right dot. If they are the same, assign that position the same color as the dot directly above. If they differ, assign that location the opposite color of the dot directly above. What does the next row look like?

{*Self-check: there should be 11 black dots.*}

Can you think of a rule that generates the pattern on the right?

{*Hint: it depends on the number of colors amongst the dots above, above-right, and above-left.*}

(5) Consider the following table:

10	9	8	7	6	...
12	11	10	9	8	...
14	13	12	11	10	...
16	15	14	13	12	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

The value in row 2, column 4 is a 9. Give a formula for the value in row i, column j.

(6) Given integers $k \geq 0$ and $m > 0$, k $(mod\ m)$ denotes the integer remainder that's left over when you divide k by m. For example, 11 divided by 4 is 2 with a remainder of 3, so 11 $(mod\ 4)$ is 3. What are the values of 15 $(mod\ 7)$, 30 $(mod\ 4)$ and 6 $(mod\ 10)$?

{*Self-check: your answers should sum to 9.*}