Continuation-Passing Style
Remember accumulator-passing style? Another strategy for producing tail recursions is *Continuation-passing style*. The continuation of an expression is what we do with the result of that expression. For example, the continuation of the expression (+ 3 4) in (* 2 (+ 3 4)) is that we multiply it by 2. We represent continuations in Scheme as functions of 1 variable, where the variable represents the result of the expression. The continuation of (+ 3 4) in the expression (* 2 (+ 3 4)) is (lambda (y) (* 2 y)).
In continuation-passing style, the recursive functions carry their continuations around as an extra parameter. Here is an example that sums the elements of a vector:

```scheme
(define sum-k
  (lambda (vec  k)
    (cond
      [(null? vec) (k 0)]
      [else (sum-k (cdr vec) (lambda (y) (k (+ y (car vec))))))]))

At the top level the continuation is (lambda (x) x). So
(sum-k '(1 2 3 4) (lambda (x) x)) is 10.
```
(define sum-k
  (lambda (vec k)
    (cond
      [(null? vec) (k 0)]
      [else (sum-k (cdr vec) (lambda (y) (k (+ y (car vec)))))])))

The interesting line is the else condition of the cond expression. We do a tail-recursive call on the cdr of vec. That much isn't surprising. The new continuation is lambda (y), where y represents the answer to (sum-k (cdr vec)...). We take y and add (car vec) to it; this gets us the sum of vec. We then apply k, the incoming continuation to this, because k tells us what to do with the answer.
Note that the top-level continuation, which we give at the start of the computation, is usually \( \lambda (x) x \): This means "return the answer".
Here are rules for writing continuation-passing style functions:

• Continuations are represented by functions of one argument. You can think of this argument as the result of the recursive call.
• At the top level the continuation is always the identity: 
  \( (\text{lambda } (y) y) \)
• Every recursive function gets an additional argument, \( k \), which is the continuation for a call to this function.
• The continuation parameter must be applied to any answer produced by the function -- instead of returning \( x \) we return \( (k \ x) \)
• All recursive calls are tail-recursive. Context gained during evaluation of the function is incorporated in the new continuation passed in the recursive call.
Here is a reverse function done in continuation-passing style:

```
(define rev-k
  (lambda (lat k)
    (cond
      [(null? lat) (k null)]
      [else (rev-k (cdr lat) (lambda (y) (k (append y (list (car lat))))))])))
```

The continuation for the recursive call is \((\text{lambda} \ (y)\ ... )\) so \(y\) will be the reversal of \(\text{cdr} \ \text{lat}\), append \(y\) onto the list whose only element is \((\text{car} \ \text{lat})\) -- this gets us the reversal of \(\text{lat}\) -- and apply the incoming continuation to this result.
One more example. The append function joins together two lists: (append '(1 2 3) '(4 5 6)) is (1 2 3 4 5 6).

(define append-k (lambda (lat1 lat2 k)  
    (cond  
        [(null? lat1) (k lat2)]  
        [else (append-k (cdr lat1) lat2 (lambda (y) (k(cons (car lat1)y))))]]))

The continuation for the recursive call says "Let y be the result of appending (cdr lat1) onto lat2. cons (car lat1) onto y, and apply the incoming continuation to the result."
Programming with explicit continuations gives you a lot of control. Here is one example of this. Consider a simple recursive function that sums the elements of a vector.

```scheme
(define sum
  (lambda (vec)
    (cond
      [(null? vec) 0]
      [else (+ (car vec) (sum (cdr vec)))])))
```

Suppose we want to modify this to return 'error if we get to an element of vec that isn't a number.
The following doesn't work:

```
(define sum1
  (lambda (vec)
    (cond
      [(null? vec) 0]
      [(not (number? (car vec))) 'error]
      [else (+ (car vec) (sum1 (cdr vec)))])))
```

If we call this with a bad "vector": (sum1 '(1 2 bob), then when we recurse to (sum1 '(2 bob)) we want to add 2 to (sum1 '(bob)). However, (sum1 '(bob)) is 'error, and we can't add 2 to 'error, so our program crashes
However, since continuation-passing style uses tail-recursion, we can pass the 'error symbol back to the top. The following does work:

```scheme
(define sum-k
  (lambda (vec k)
    (cond
      [(null? vec) (k 0)]
      [(not (number? (car vec))) 'error]
      [else (sum-k (cdr vec) (lambda (y) (k (+ y (car vec))))))])))
```

Now `(sum-k '(1 2 3) (lambda (x) x))` is 6

`(sum-k '(1 2 bob) (lambda (x) x))` is 'error
This is even better done by giving an explicit *error continuation*, which tells us what to do with an error:

```scheme
(define sum3
  (letrec ([sum-k (lambda (vec k err)
                   (cond
                     [(null? vec) (k 0)]
                     [(not (number? (car vec))) (err (car vec))]
                     [else (sum-k (cdr vec)
                                  (lambda (y) (k (+ y (car vec))))
                                  err)]])])
   (lambda (vec) (sum-k vec
                  (lambda (x) x)
                  (lambda (x) (list 'bad-element x))))))
```
Now (sum3 '(1 2 3)) is 6 and (sum3 '(1 2 bob)) is (bad-element bob).