More About Scheme:
closures, environments, set! and let
There is a global environment that binds symbols to values.

The expression

   (define foo bar)

creates (or changes) the binding of symbol foo to the value of bar.

Consider this example:

   (define f (lambda (x) (+ x 1)))
   (define g (lambda (x) (f (* x 5)))
   (g 2) => 11

   (define f (lambda (x) 23)
   Now (g 2) => 23

**Moral**: Be careful how you define things. (define foo bar) can change things other than foo.
Procedures in Scheme are "first-class values". This means we can use procedures like any other data value -- they can be arguments to calls, or the return values of other procedures. To understand this we need to be very explicit about how procedure calls work in Scheme.

When we evaluate a list such as (A B C) each of the elements is evaluated in the current environment, the first element should evaluate to a procedure, and that procedure is called with the values of the other elements of the list. For the call a new environment is created in which the parameters of the procedure are bound to the values of the arguments, and the body of the procedure is evaluated in this environment.
Example:

```
(define foo (lambda (x y) (+ (* x 2) y)))
(define bar (lambda (x) (foo 3 x)))
```

Evaluate (bar 7) in the top-level environment.

The top-level environment has symbols foo and bar bound to the values of their procedures.

We evaluate bar and get `<procedure bar>`; we evaluate 7 and get 7. So we call `<procedure bar>` with value 7. A new environment is created with x, the parameter of bar, bound to 7. We evaluate the body of bar: (foo 3 x) in this environment. Another environment is created with bindings for the parameters of foo: x is bound to 3 and y is bound to 7. We evaluate (+ (* x 2) y) in this environment and get 13.
To make this possible, procedures need to carry their environments with them. The value of a procedure consists of three things:

a) The parameter list of the procedure.
b) The environment in which the procedure was created.
c) The body of the procedure.

This is called a closure. When we apply the procedure to arguments, the environment of its closure is extended to include bindings of the parameters to the values of the arguments, and the body is evaluated within this extended environment.
Here is another example:

\[
\text{(define fo (lambda (x) (lambda (y) (+ x y)))})
\]
Evaluate \((\text{fo} \ 3 \ 4)\) in the top-level environment.

The top-level environment binds \(\text{fo}\) to its closure. The environment for \#<procedure \text{fo}> is the top-level. The first thing we do is evaluate \((\text{fo} \ 3)\). This extends \(\text{fo}\)'s environment (the top level) with a binding for \(x\) to 3. We then evaluate \(\text{fo}\)'s body: \((\text{lambda} (y) (+ x y))\) in this environment. This evaluates to a closure with parameter list \((y)\), environment \{\(x->3\}\}, and body \((+ x y)\). So that is the value of \((\text{fo} \ 3)\). We then apply this closure to the argument 4. To do this we extend the closure's environment to include a binding of its parameter \(y\) to 4: the new environment is \{\(x->3, \ y->4\}\}. We evaluate \((+ x y)\) in this environment and get 7.
Note the similarity and differences between the following functions:

(\(\text{define } f (\lambda (x\ y) (+ x y))\))

(\(\text{define } \text{fo} (\lambda (x) (\lambda (y) (+ x y)))\))

They are similar, since \((f 3 4)\) is 7 and \((\text{fo} 3) 4\) is 7. But they are also different since \(f\) is a function of two variables and \(\text{fo}\) is a function of 1. You can think of \((\text{fo} 3)\) as the result of freezing the first argument of \(f\) as \(x=3\), leaving a function of just variable \(y\). This is called \textit{currying} \(f\) (named after Haskell Curry, a mathematician at Penn State from 1929 to 1965 who studied such things. Mr. Curry, by the way, was so important to the foundations of programming languages that long after his death the Haskell language was named after him).
Here are some new Scheme expressions: begin, set! and let.

\[(\text{begin } e_1 e_2 e_3 e_4 \ldots e_n)\]
This evaluates each of the expressions, returning the result of the last one.

\[(\text{begin } (+ 3 4) (* 5 6)) \Rightarrow 30\]

Of course, this is a bit silly until we get expressions that have side-effects.
(set! x e) (pronounced "set bang")
This changes the binding of symbol x in the current environment to the value of e. Symbol x must already be bound in the environment.

This gives us side-effects:

(define x 0)
(begin (set! x 5) (* x 3) => 15
set! causes all sorts of problems that we will talk about later. As much as possible we will program functionally -- without set!
This evaluates the binding values exp1, exp2, etc. in the current environment. A new environment is created extending the current environment with bindings for each of the symbols sym1, sym2, ... to the value of its expression. The value of the let expression is the value of its body evaluated within this new environment. The body may be either a single expression or a sequence of expressions. In the latter case each of these is evaluated and the value of the last of them is returned.
Examples:

(let ([x 3] [y 4]) (+ x y))  => 7

(let
    ([f  (lambda (x) (+ x 1))]
     [g  (lambda (x) (* 2 x))])
  (f (g 3) )  => 7

(define sumSquares
  (let ([sq (lambda (x) (* x x))])
    (lambda (lat)
      (cond
        [(null? lat) 0]
        [else (+ (sq (car lat)) (sumSquares (cdr lat))))])))
Compare these very similar functions. Both produce the correct sum of the squares of the numbers in \textit{lat}. If \textit{lat} is \((1 \ 2 \ 3)\) both return 14.

\begin{verbatim}
(define sumSquares
  (let ([sq (lambda (x) (* x x))])
    (lambda (lat)
      (cond
        [(null? lat) 0]
        [else (+ (sq (car lat)) (sumSquares (cdr lat)))]))))

(define sumSquares2 (lambda (lat)
  (let ([sq (lambda (x) (* x x))])
    (cond
      [(null? lat) 0]
      [else (+ (sq (car lat)) (sumSquares2 (cdr lat)))]))))
\end{verbatim}
sumSquares and sumSquares2 are both recursive procedures. The closure environment for sumSquares contains a binding for procedure sq. The closure environment for sumSquares2 is the top-level. Each time we call sumSquares2 its body, containing the let-binding for sq, has to be evaluated. This doesn't happen when we call sumSquares, so sumSquares is more efficient than sumSquares2.