Another Undecidability Example

Let $L_{101}$ be the set of encodings of TMs that accept the string 101 and no other string. Is $L_{101}$ Recursively Enumerable?

Answer: No. Reduce complement of $L_u$ to it.

Given $(M, w)$ we create $M'$. $M'$ takes input $x$. If $x$ is 101, $M'$ accepts $x$. If $x$ is not 101 $M'$ ignores $x$ and simulates $M$ on $w$, accepting $x$ if $M$ accepts $w$.

If $M$ accepts $w$, $M'$ accepts all strings. If $M'$ does not accept $w$, $M'$ accepts only 101.

A recognizer for $L_{101}$ will recognize if $M$ does not accept $w$. Thus, a recognizer for $L_{101}$ creates a recognizer for the complement of $L_u$, and we know that can’t exist.