A Field Guide to the Chomsky Construction of a grammar from a PDA

**Notation:** \( [pXq] \) will generate all strings \( w \) so that \( (p,w,X)^* \Rightarrow (q,e,e) \)

i.e., \([pXq]\) represents all strings that take the PDA from state \( p \) to state \( q \) while popping \( X \) off the stack.

**Rule 1:** \( S \Rightarrow [QZ_0p] \) where \( Q \) is the start state, \( p \) is any state

**Rule 2:** Suppose the PDA has transition

\[
\begin{array}{c}
\text{q} \\
\downarrow \text{a,X|Y_1...Y_k} \\
\text{r}
\end{array}
\]

Then for every sequence of \( k \) states \( r_1...r_k \)

\( [qXr_k] \Rightarrow a[rY_{r_1}][r_1Y_{r_2}] ... [r_{k-1}Y_{r_k}] \)

**Rule 3:** If there is a transition

\[
\begin{array}{c}
\text{q} \\
\downarrow \text{a,X|e} \\
\text{r}
\end{array}
\]

then \( [qXr] \Rightarrow a \)
**Rule 4:** Suppose there is a transition

\[
\begin{array}{c}
\text{q} \\
\overset{\epsilon, X|Y_1\cdots Y_k}{\longrightarrow}
\end{array}
\begin{array}{c}
r
\end{array}
\]

then for any sequence of states \( r_1..r_k \)

\[ [qXr_k] \Rightarrow [r_1 r_1][r_2 r_2] \cdots [r_{k-1} r_k] \]

**Rule 5:** Suppose there is a transition

\[
\begin{array}{c}
\text{q} \\
\overset{\epsilon, X|\epsilon}{\longrightarrow}
\end{array}
\begin{array}{c}
r
\end{array}
\]

then there is a rule \( [qXr] \Rightarrow \epsilon \)