1. Which languages are regular? You don’t need to prove your answers. Write an “R” in the blank next to the description of each language you think is regular. Write “N” for any language you think is not regular. In each case the alphabet is $\Sigma = \{0, 1\}$

   a. ______Strings that end in exactly five 1s. So 01011111 is in this language but 010111111 is not.

   b. ______Strings with any number of 0s followed by an even number of 1s.

   c. ______$\{0^m1^n \mid$ if $m$ is even then $n$ is also even; if $m$ is odd then $n$ is also odd$\}$

   d. ______Strings where the digits sum to a number divisible by 5 (i.e., the digits sum to 0, 5, 10, 15, etc.)

   e. ______Strings where there are at least as many 0s as 1s.

   f. ______$0^* \mathcal{L}$ where $\mathcal{L} = \{0^n \mid n \text{ is prime}\}$. Note that strings in this language have any number of 0s followed by a prime number of 0s.
2. Give a DFA for the strings of 0s and 1s that contain the substring 010. For example, 110101 should be accepted by this DFA but 1001100 should not be accepted.
3. Here is an $\varepsilon$-NFA, with start state A.
   a) Convert this NFA to a DFA
   b) Describe in English the strings it accepts.
4. Suppose we know that for some language $\mathcal{L}$ the language $00\mathcal{L} = \{00\alpha \mid \alpha \in \mathcal{L}\}$ is regular. Must $\mathcal{L}$ be regular? Either give an example where $\mathcal{L}$ is not regular and $00\mathcal{L}$ is regular, or else show that $\mathcal{L}$ must be regular if $00\mathcal{L}$ is.
5. Consider the following DFA. We had an algorithm for converting a DFA to a regular expression. This involved making a table of regular expressions $r_{ij}^k$.

Here is the first column of a table of the $r_{ij}^k$ expressions; find the 4 entries of the second column.

<table>
<thead>
<tr>
<th></th>
<th>$k=0$</th>
<th>$k=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{11}^k$</td>
<td>$\epsilon + 1$</td>
<td></td>
</tr>
<tr>
<td>$r_{12}^k$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$r_{21}^k$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$r_{22}^k$</td>
<td>$\epsilon + 0$</td>
<td></td>
</tr>
</tbody>
</table>
6. Use the pumping lemma to show carefully that the language \( \{0^m1^n0^n | m \geq 2, n \geq 0\} \) is not regular.
This page is extra space. If you want me to grade anything here indicate that clearly.

Please write and sign the Honor Pledge when you have finished the exam.